

Hong Kong Mathematics Olympiad (2022/23)
 Finals (Individual – Event 1)

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>	Team No.	<input type="text"/>
			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
						<input type="text"/>		

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. If A is the unit digit of 2023^{2024} , find the value of A .

若 A 是 2023^{2024} 的个位数，求 A 的值。

$A =$

2. If B is the number of positive common factors of 336^A and 528^A , find the value of B .

若 B 是 336^A 和 528^A 的正公因子的数量，求 B 的值。

$B =$

3. A 3×3 grid is partially completed as shown below. Fill each square of the grid with a positive integer such that the sum of the three numbers in each row, column, and both main diagonals are equal. Find the value of C .

下图是一个未完成的九宫格，每一格须填入一个正整数使得每一行、每一列和两个主对角在线的三个数字总和相等。求 C 的值。

C	16	$2B$
4		

$C =$

4. $\frac{C}{2}$ couples are attending a party, which means that there are C people present. At this party, no one will shake hands repeatedly with the same guest. The party also has the condition that each husband will shake hands with every guest except his own wife, and wives will shake hands with every guest except other wives. D represents the total number of handshakes between the C people at the party. Find the value of D .

有 $\frac{C}{2}$ 对夫妇参加了一个派对，即在派对上共有 C 人。在这个派对上，没有人会和同一位客人重复地握手。此外，每位丈夫都会和他妻子以外的所有客人握手，而妻子们不会与其他妻子握手，但会和其他客人握手。 D 是在这派对上 C 人之间握手的总数，求 D 的值。

$D =$

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 Finals (Individual – Event 2)

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			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
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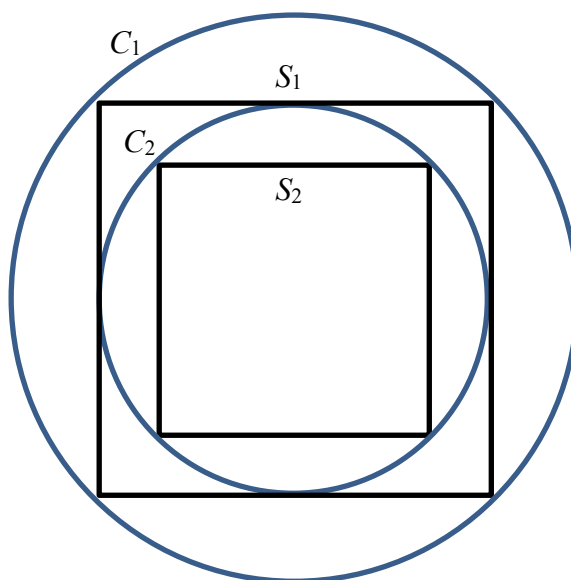
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特别声明，答案须用数字表达，并化至最简。

1. Find the smallest positive integer α that is divisible by 11 and the sum of its digits is equal to 38.
找出一个能被 11 整除，且各数字之和是 38 的最小正整数 α 。

$\alpha =$

2. Let α' be the last digit of α . A circle C_1 of radius α' circumscribes a square S_1 which inscribes a circle C_2 . C_2 circumscribes square S_2 and so forth indefinitely. Find the area β of the square S_6 .
设 α 的最后一位数字是 α' 。 C_1 是正方形 S_1 的外接圆，它的半径为 α' ， C_2 是正方形 S_1 的内接圆， C_2 亦是正方形 S_2 的外接圆，如此类推。求正方形 S_6 的面积 β 。



$\beta =$

3. Let $[\beta]$ be the integral part of β . The equation $x^3 + nx^2 + mx + [\beta] = 0$, where m, n are integers, has three integer roots. Suppose that the roots are not all positive, if $\gamma = n - m$, find γ .

设 β 的整数部分是 $[\beta]$ 。若 m, n 为整数，方程 $x^3 + nx^2 + mx + [\beta] = 0$ 有三个整数根。假设这三个根不全是正整数，若 $\gamma = n - m$ ，求 γ 。

$\gamma =$

4. On the xy -plane, a move consists of independently increasing (or decreasing) x -coordinate and y -coordinate by 1 (i.e. moving diagonally). If δ is the number of ways to start from $(0, 0)$, make exactly 12 moves and end at (γ, γ) , find the value of δ .

在 xy 平面上，每一步移动都包含 x 坐标和 y 坐标分别地增加 (或减少) 1 个单位 (即对角线移动)。若 δ 是由 $(0, 0)$ 开始只行走 12 步后到达 (γ, γ) 的方法，求 δ 。

$\delta =$

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 Finals (Individual – Event 3)

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			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
						<input type="text"/>		

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除非特别声明，答案须用数字表达，并化至最简。

1. Given that m and n are positive integers. If $m+n+mn=76$ and $A=m+n$, find the value of A .

已知 m 和 n 均为正整数。如果 $m+n+mn=76$ 及 $A=m+n$ ，求 A 的值。

$A =$

2. If $B = \sqrt{(401)^2 - 100A}$, find the value of B .

如果 $B = \sqrt{(401)^2 - 100A}$ ，求 B 的值。

$B =$

3. The area of the rhombus on the xy -plane with vertices $(B+1, 0)$, $(-B-1, 0)$, $(0, 1)$ and $(0, -1)$ is C square units. Find the value of C .

在 xy 平面上由点 $(B+1, 0)$, $(-B-1, 0)$, $(0, 1)$ 及 $(0, -1)$ 所组成之菱形的面积为 C 平方单位，求 C 的值。

$C =$

4. If D is a positive integer such that $\left(\frac{C}{4} + 56\right)^{\frac{1}{D}} = D$, find the value of D .

如果 D 是正整数且 $\left(\frac{C}{4} + 56\right)^{\frac{1}{D}} = D$ ，求 D 的值。

$D =$

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 Finals (Individual – Event 4)

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Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
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			Total score			<input type="text"/>

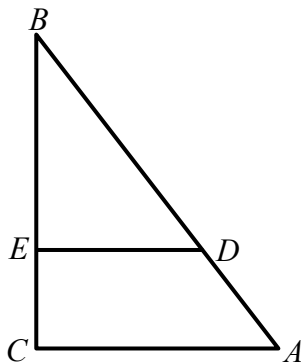
Team No.	<input type="text"/>
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除非特别声明，答案须用数字表达，并化至最简。

1. In triangle ABC , $\angle C = 90^\circ$, $DE \perp BC$, $BE = AC$, $BD = \frac{1}{2}$ and $DE + BC = 1$. If $\alpha = 4ED$, find α .

在三角形 ABC 中， $\angle C = 90^\circ$ ， $DE \perp BC$ ， $BE = AC$ ， $BD = \frac{1}{2}$ 及 $DE + BC = 1$ 。如果 $\alpha = 4ED$ ，求 α 。



$\alpha =$

2. If $f(a) = a - 2$, $F(a, b) = b^2 + a + \alpha$ and $\beta = F(3, f(4))$, find β .

若 $f(a) = a - 2$ ，且 $F(a, b) = b^2 + a + \alpha$ 及 $\beta = F(3, f(4))$ ，求 β 。

$\beta =$

3. If the system of equations

$$\begin{cases} x^2 - 3xy + 2y^2 - z^2 = 31 \\ -x^2 + 6yz + 2z^2 = 44 \\ x^2 + xy + \beta \cdot z^2 = 100 \end{cases}$$

has γ sets of integer solutions, find γ .

如果方程组

$$\begin{cases} x^2 - 3xy + 2y^2 - z^2 = 31 \\ -x^2 + 6yz + 2z^2 = 44 \\ x^2 + xy + \beta \cdot z^2 = 100 \end{cases}$$

整数解的个数是 γ ，求 γ 。

$\gamma =$

4. In a triangle ABC , $AB = AC$, $\angle A = 40^\circ + \gamma^\circ$. Point O is inside the triangle ABC with $\angle OBC = \angle OCA$. If $\angle BOC = \delta^\circ$, find δ .

在三角形 ABC 中， $AB = AC$ ， $\angle A = 40^\circ + \gamma^\circ$ 。点 O 在三角形 ABC 内且 $\angle OBC = \angle OCA$ 。如果 $\angle BOC = \delta^\circ$ ，求 δ 。

$\delta =$